Implementation of the Unscented Kalman Filter and a simple Augmentation System for GNSS SDR receivers

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BIOGRAPHY

Roberto Capua is responsible for GNSS R&D activity at Sogei S.p.A. He worked in several GNSS R&D and EC Framework Projects for Galileo applications and serves as a delegate to the Galileo Services consortium. His areas of activity are: advanced high-accuracy GNSS and Augmentation systems, GNSS SDR development and GNSS surveying.

Antonio Bottaro is currently the Head of the Research and Development Department within Sogei S.p.A. His main area of activity concerns Advanced GIS and GNSS platforms Design and Development.

ABSTRACT

The paper presents the results of the development of advanced PVT solutions for the Sogei's GNSS SDR. Such Real-Time platform is based on an internally designed Front-End and is totally running on a General Purpose Notebook.

The EKF deals with non-linear systems through process and observations functions linearization. Non-linearity errors can lead to poor Kalman filter performances or to divergence.

Linearization does not allow correct state and measurements Covariance matrices propagation in presence of significant dynamic conditions. Jacobians introduce severe computational burdens and cannot be applied in the presence of discontinuities and singularities.

In order to overcome such limitations, an Unscented Kalman Filter (UKF) algorithm has been developed and tested within the SDR.

The UKF avoids linearization. Non-linear transformation is performed at each step on a set of deterministic sigma points extracted from the Covariance matrix, allowing prediction errors minimization. Dependence on initial conditions is reduced.

Due to inherently quantized measurements and computational load constraints, SDR can highly benefit from the use of an UKF.

In our implementation, a numeric efficient GPS L1 pseudoranges and Doppler measurements representation has been developed. A fine tuning phase has been carried out in mixed static and dynamic conditions. A test campaign has been carried out on a car equipped with the GNSS SDR. True trajectory was determined through RTK. An accuracy of 5m and improved robustness in case of abruptly changing routes is achieved.

For SDR, a simple and inexpensive augmentation technique for medium level accuracy is advisable. A Local Augmentation based on Coordinates Corrections method has been implemented. Corrections are calculated at SDR rover side, using Reference Station measurements acquired through NTRIP and RTCM messages. An accuracy of 2 m is reached. Such a solution can pave the way for implementing reliable and low cost Augmentation systems totally based on SDR receivers.

INTRODUCTION

Governmental applications are often requiring high performances (high accuracy, high reliability and high continuity, security), while needing Open and customizable technologies. Such technologies should be also able to incorporate legacy systems and proprietary data formats, in order to facilitate their introduction with minimal impact on well consolidated workflows and regulation frameworks.

Sogei, the ICT Company of the Ministry of Economy and Finance of Italy, started in 2008 a Project intended to develop a GNSS SDR receiver for high demanding applications.

Applications of interest for the project are Land Administration and Land Registry/Cadastral Maps updating through high precision GNSS surveying, Border Controls and Dangerous Goods Tracing and Tracking for Customs operations.

High accuracy applications require the installation and maintenance of extensive GNSS Reference Stations Networks at national or regional level. Such Reference Stations are costly in terms of upgrade and maintenance.

On the other hand, Freights management applications for institutional applications are requiring a high level of reliability, data security and output customizability.

For such kind of applications, GNSS SDR is one of the most suitable cutting-edge technologies, due to its intrinsic flexibility, scalability and openness to the integration of new signal waveforms.

Sogei GNSS SDR platform is able to perform Real Time GPS, SBAS and GIOVE Acquisition and Tracking. The architectural components of the system are:

- *GNSS IF Front-End*: internally designed and totally based on COTS components and the Maxim MAX2769 sampler, outputs IF samples on a USB
- *Acquisition, Tracking and PVT software modules*: implemented in C/C++, they are multi-threaded and communicating through TCP/IP sockets
- *Quad-Core Notebook*: running the GNSS SDR Software
- *GUIs*: implemented using OpenSource wxWidgets and RTKLIB tools

Multi-Core programming is implemented using the Cross-Platform OpenMP Parallel Programming API, while high computational tasks like FFT and multiplications are performed using SIMD (MMX and SSE) instructions.

In Figure 1, the Sogei GNSS platform Architecture is represented.

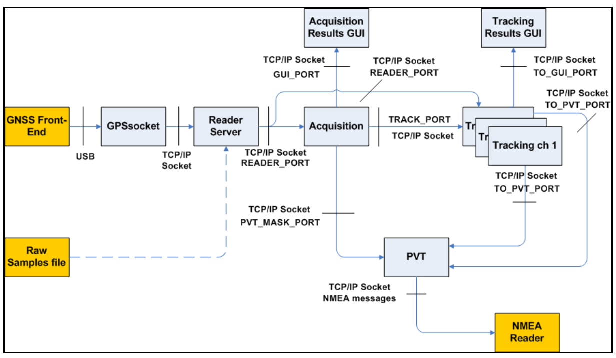

Figure 1 - Sogei GNSS Platform System Architecture

IF is set by default at 1.023 MHZ with a sampling frequency of 4.092 MHz for GPS and a 2-bits (1-bit Sign and 1-bit Magnitude) quantization. The integration of Galileo can be achieved, using an IF of 2.046 MHz and a sampling frequency of 8.184 MHz.

Acquisition module performs a Parallel Code Phase Search. Sine and cosine Look-Up Tables and FFT code replicas are generated off-line and loaded into the cache memory at start-up. Acquisition is performed in two steps (Coarse and Fine), in order to provide a finer Doppler estimate. Warm start and Hot start are available.

Tracking Loop is implemented by default through a second order PLL and a first order FLL. A third-order PLL can be activated on request. The FLL is selectively activated at start-up or after a loss of lock and switched off after a positive test on phase error variance. In order to improve performances through data parallelism, raw samples are packet in two words of 32 Sign and 32 Magnitude bits. Code correlation and Carrier mixing is therefore implemented through XOR.

In order to minimize the number of needed correlators, the normalized Dot product DLL discriminator is used ([R4]) for Code tracking.

Prompt and Early-Minus-Late Code Look-Up tables are generated off-line for 32 Code phase steps. Such tables are oversampled, due to the fact that the sampling frequency is higher than the chip rate. Code Doppler is assumed to be zero in the Code-Look-Up table.

Sine and Cosine Intermediate Frequency Look-Up tables for carrier wipe-off are generated with $\pi/8$ phase steps and 175 Hz frequency steps. Carrier levels are represented by 2 bits (Sign and Magnitude).

Costas loop discriminator is implemented through the classical $\mathrm{atan}(Q_P/I_P)$, while FLL uses the atan2 function.

The initial PVT was based on an Extended Kalman Filter (EKF). It is able to calculate navigation solution into NMEA GGA message with 500 ms update rate. Details about the PVT implementation will be described in the following paragraphs.

EXTENTED KALMAN FILTER BASICS AND DRAWBACKS

Let us give a non-linear discrete system with equations:

$$
\begin{aligned}
x(k+1) &= f\left[x(k), u(k), v(k), k\right] \sim \mathrm{N}(0, \mathrm{Q}_k) \\
z(k) &= h\left[x(k), u(k), k\right] + w(k) \sim \mathrm{N}(0, \mathrm{R}_k)
\end{aligned}
\qquad (1)
$$

EKF prediction is performed using Taylor series expansion:

$$\hat{x}(k+1) = f\left[\hat{x}(k \mid k), u(k), k\right]$$
$$P(k+1 \mid k) = \Im_F P(k \mid k) \Im_F^T + Q(k+1) \quad (2)$$

where $\Im_F$ is the Jacobian matrix of the state transition function f.

Kalman filter update equations are:

$$K_{k+1} = P(k+1 \mid k) H^T (HP(k+1 \mid k) H^T + R_k)^{-1}$$
$$\hat{x}(k+1 \mid k+1) = \hat{x}(k+1 \mid k) + K_{k+1}(z_k - Hx(k \mid k)) \quad (3)$$
$$P(k+1 \mid k+1) = P(k+1 \mid k-1) - K_{k+1} HP(k \mid k-1)$$

where K_{k+1} is the Gain matrix, P(k+1|k+1) is the updated State Covariance matrix and:

$$H = \left.\frac{\partial h}{\partial x}\right|_k \quad (4)$$

is the Jacobian of the measurement function h.

As can be seen, classical Kalman Filter can be simply implemented using basic algebraic functions.

Unfortunately, some drawbacks that lead to inefficiency and possible filter divergence are present:

- Matrix inversion and Jacobians calculation for Gain and Design Matrix calculation imply high computational load
- Prediction equations provide very poor approximation of the true mean and Covariance matrix due to non-linearities
- Good quality initial conditions are needed for assuring filter convergence

Matrix inversion computational load can be reduced assuming pseudorange and Doppler measurements decorrelated among satellites, but recalculation of Jacobians elements at each epoch is still needed.

EKF prediction can fail due to linearization. This can be seen for instance in a circular motion of a vehicle with constant velocity (see Figure 2 and [R5]). As can be seen, the Covariance is linearly projected in the initial direction of travel, leading to loose the correct orientation (the largest component of the uncertainty at k+1 step is not in the current direction of travel).

Furthermore, due to the linearization, EKF cannot be applied with non differentiable functions.

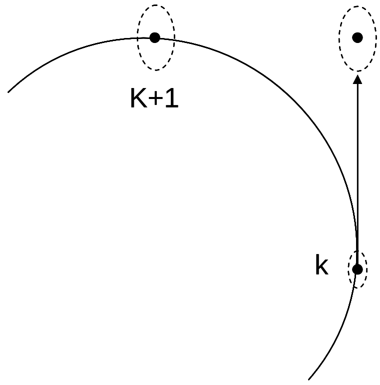

Figure 2 – EKF Covariance propagation example

For better estimating true mean and Covariance matrix, as well as reducing computational load, the Unscented transformation is used, as introduced in the following paragraphs.

UNSCENTED KALMAN FILTER BASICS AND ADVANTAGES

The Unscented Kalman Filter tries to overcome the limitations of Extended Kalman Filter. It starts (e.g. [R4]) from the consideration that it is easier to approximate a Gaussian distribution through a fixed number of parameters than a non linear function.

In order to represent a given distribution with mean ($\bar{x}$) and State Covariance matrix, denoted as P_{xx} for highlighting relevant terms, we can use the following 2n+1 points named *Sigma Points*:

$$\chi_0 = \bar{x}$$
$$\chi_i = \bar{x} + \sigma_i \quad i = 1,..,2n$$

where $\bar{x}$ is the mean and σ_i are taken from rows or columns of the matrices $\pm\sqrt{(n+k)P_{xx}}$. The last multiplication factor is due to the need of respecting the property:

$$P_{xx} = \frac{1}{2(n+k)} \sum_{i=1}^{2n} [\chi_i - \bar{x}][\chi_i - \bar{x}]^T$$

k is a scaling parameter to be chosen for scaling the error in the approximation of the fourth order moment. It can be demonstrated that choosing n+k=3, it is possible to minimize the fourth order moment with respect to a Gaussian distribution ([R4]).

Having a non linear transformation y=g(x), we can therefore work in the χ domain and transform single sigma points, as showed in Figure 3:

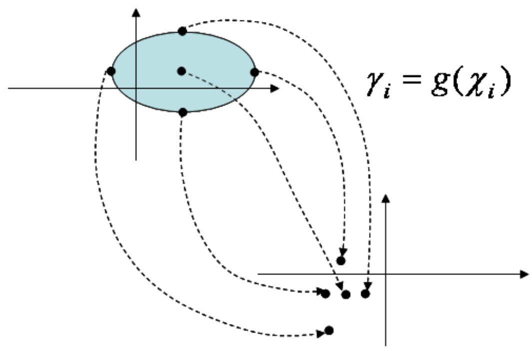

Figure 3 – Sigma points transformation

The predicted mean is computed as:

$$\bar{y} = \frac{1}{n+k}\left\{k\gamma_0 + \frac{1}{2}\sum_{i=1}^{2n}\chi_i\right\}$$

and the predicted Covariance is:

$$P_{yy} = \frac{1}{n+k}\left\{k[\gamma_0 - \bar{y}][\gamma_0 - \bar{y}]^T + \frac{1}{2}\sum_{i=1}^{2n}[\gamma_i - \bar{y}][\gamma_i - \bar{y}]^T\right\}$$

Such representation by points allows to correctly predict mean and Covariance matrix, as in the case of the previous example (see Figure 4).

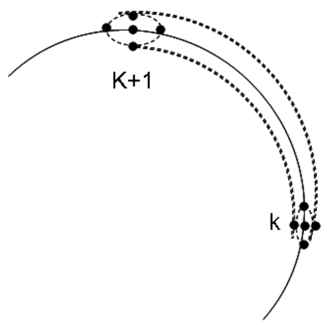

Figure 4 – Unscented covariance propagation example

The UKF algorithm used for the implementation of the GNSS SDR PVT is a *scaled* version of the classical UKF. The scaling is used for modulating the spread of the sigma Points around the mean.

In the following, the Unscented Kalman Filter estimation process is reported.

1. Initialization and parameters definition:

$$\bar{x}(0), P_{xx}(0)$$

$$\bar{k} = \alpha^2(n+\tilde{k}) - n$$
$$M(0) = \sqrt{(n+\bar{k})[P(0)+Q(0)]}$$
$$S(0) = [0 \quad | \quad -M(0) \quad | \quad M(0)]$$
$$\chi_i(0\,|\,0) = \hat{x}(0) + \sigma_i(0) \qquad \begin{matrix} i = 0,..,2n \\ \sigma_i(0) = i^{th} \quad column \quad of \quad S(0) \end{matrix}$$

Cholesky decomposition is used for calculating the square root matrix M.

2. Mean and State Covariance matrix instantiation and prediction:

$$\chi_i(k\,|\,k) = [\hat{x}(k\,|\,k), \hat{x}(k\,|\,k) \pm \sigma_i(k)]$$
$$\chi_i(k+1\,|\,k) = f(\chi_i(k\,|\,k), u(k), k)$$
$$\hat{x}(k+1\,|\,k) = \sum_{i=0}^{2n} w_i^m \chi_i(k+1\,|\,k)$$
$$P(k+1\,|\,k) = \sum_{i=0}^{2n} w_i^c [\chi_i(k+1\,|\,k) - \hat{x}(k+1\,|\,k)][\chi_i(k+1\,|\,k) - \hat{x}(k+1\,|\,k)]^T$$

where:

$$\bar{k} = \alpha^2(n+\tilde{k}) - n$$
$$w_0^m = \frac{\bar{k}}{n+\bar{k}}$$
$$w_0^c = \frac{\bar{k}}{n+\bar{k}} + (1-\alpha^2+\beta)$$
$$w_i^c = w_i^m = \frac{1}{2(n+\bar{k})} \qquad i = 1,..,2n$$

3. Measurements instantiation and prediction:

$$\zeta_i(k+1\,|\,k) = h(\chi_i(k+1\,|\,k), u(k), t)$$
$$\hat{z}(k+1\,|\,k) = \sum_{i=o}^{2n} w_i^m \zeta_i(k+1\,|\,k)$$

4. Innovation and Cross-Correlation Covariance matrices calculation:

$$P_{vv}(k+1\,|\,k) = \sum_{i=0}^{2n} w_i^c [\zeta_i(k\,|\,k-1) - \hat{z}(k+1\,|\,k)][\zeta_i(k\,|\,k-1) - \hat{z}(k+1\,|\,k)]^T + R(k+1)$$
$$P_{xz}(k+1\,|\,k) = \sum_{i=0}^{2n} w_i^c [\chi_i(k\,|\,k-1) - \hat{x}(k+1\,|\,k)][\zeta_i(k\,|\,k-1) - \hat{z}(k+1\,|\,k)]^T + R(k+1)$$

where P_{xz} is the cross-correlation matrix.

5. Classical Kalman Update:

$$K(k+1) = P_{xz}(k+1\,|\,k)P_{vv}^{-1}(k+1\,|\,k)$$
$$\hat{x}(k+1\,|\,k) = \hat{x}(k\,|\,k) + K(k+1)(z(k) - \hat{z}(k+1\,|\,k))$$
$$P(k+1\,|\,k+1) = P(k+1\,|\,k) - K(k+1)P_{vv}(k+1\,|\,k)K^T(k+1)$$

In the previous equations, $\bar{k}$ is the scaling parameter, α determines the spread of the sigma points around the mean and it is usually set to a small value (e.g. 1e-3), $\tilde{k}$ is a secondary scaling factor and it is usually set to zero, while β is a tertiary scaling factor used for incorporating a priori knowledge about the distribution of x and emphasizing the weight of zero-sigma points in the Covariance matrix calculation (usually set to 2 for Gaussian distribution).

As can be seen, UKF implementation does not require linearization (state transition function and measurements functions are directly applied to sigma points) and it can also work in presence of discontinuities. The prediction only consists of linear algebra operations. All such advantages are fundamental for minimizing computational load in an SDR implementation.

While the classical Kalman Filter implies the propagation of n components for the state vector and $n^2/2+n/2$ components for the Covariance matrix, the UKF requires the propagation of 2n+1 sigma points only.

Furthermore, UKF is more insensitive to initial conditions with respect to EKF. It has been demonstrated that UKF rapidly converge also in presence of an initial position error of several Kilometers.

DYNAMIC MODEL FOR KALMAN FILTER IMPLEMENTATION IN THE SDR PVT

The implemented Kalman Filters includes position, velocity, receiver clock bias and drift estimation.

State vector is defined as in the following:

$$x = \begin{bmatrix} x & y & z & | & \dot{x} & \dot{y} & \dot{z} & | & b & \dot{b} \end{bmatrix}^T$$

where x, y and z are the errors on Cartesian coordinates of the receiver, b and $\dot{b}$ are the receiver code offset and drift estimation errors.

The classical discrete-time dynamic model is:

$$x_{k+1} = \Im_k x_k + w_k, w_k \sim \mathrm{N}(0, \mathrm{Q_k})$$
$$z_k = H_k x_{k-1} + v_k, v_k \sim \mathrm{N}(0, \mathrm{R_k})$$

$$\Im_k = \begin{bmatrix} I & I\Delta t & | & 0 & | & 0 & 0 \\ 0 & I & | & 0 & | & 0 & 0 \\ - & - & | & - & | & - & - \\ 0 & 0 & | & I & | & 0 & 0 \\ - & - & | & - & | & - & - \\ 0 & 0 & | & 0 & | & I & I\Delta t \\ 0 & 0 & | & 0 & | & 0 & I \end{bmatrix}$$

$$Q_k = \begin{bmatrix} Q_{pvk} & | & 0 \\ - & - & ---- \\ & | & \\ 0 & & Q_{ck} \\ & | & \end{bmatrix}$$

$$Q_{ck} = \begin{bmatrix} S_b\Delta t + S_f \Delta t^3/2 & S_f \Delta t^2/2 \\ S_f \Delta t^2/2 & S_f \Delta t \end{bmatrix}$$

where Δt is the step time, $\Phi_{k,k-1}$ is the transition matrix, Q_k and R_k and are the process and measurement noise covariance matrices and S_b and S_f are the spectral densities related to the Allan variance parameters. H is the directional cosine vector from the receiver to the satellite.

The State evolution is represented by the following equations:

$$x_{k+1} = x_k + T \cdot \dot{x}_k + \frac{T^2}{2} w_{xk}$$
$$y_{k+1} = y_k + T \cdot \dot{y}_k + \frac{T^2}{2} w_{yk}$$
$$z_{k+1} = z_k + T \cdot \dot{z}_k + \frac{T^2}{2} w_{zk}$$
$$\dot{x}_{k+1} = \dot{x}_k + T \cdot w_{xk}$$
$$\dot{y}_{k+1} = \dot{y}_k + T \cdot w_{yk}$$
$$\dot{z}_{k+1} = \dot{z}_k + T \cdot w_{zk}$$
$$b_{k+1} = b_k + T \cdot \dot{b}_k + \varepsilon_{bk}$$
$$\dot{b}_{k+1} = \dot{b}_k + \varepsilon_{fk}$$

Such dynamic model, where velocity variations (e.g. accelerations) are modeled as white noise, is here used for minimizing the computational load. It is suitable for medium dynamics vehicles, as boat or cars.

The measurement vector is:

$$z = h(x) = \begin{bmatrix} PR \\ D \end{bmatrix}$$

where PR and D are the vectors of relative pseudorange and Doppler measurements for the observed satellites.

The relative pseudorange, used as usual for SDR computation ([R2]), for satellite i can be expressed in the following way:

$$PR^i = c(t_{off} + t_{ti} - t_{s\min})$$

where c is the velocity of light, t_{off} is a bias of 68 ms (taking into account that the travel time between a GPS satellite and the Earth ranges is in the 67-86 ms range), able to assure that all relative pseudoranges calculated in this way are positive, t_{ti} is the travel time derived by the number of samples elapsed till Subframe starts and $t_{s\min}$ represents the travel time of the reference satellite.

Classical non linear measurement model is used for pseudorange estimation:

$$PR^i = \sqrt{(x^i - x_u)^2 + (y^i - y_u)^2 + (z^i - z_u)^2} + d\rho^i +$$
$$+ c(dt^i - dT) + d_{ion_i} + d_{trop_i} + \varepsilon$$

where $d\rho^i$ are the orbital errors, dt^i is the satellite clock error, dT is the receiver clock error, d_{ion_i} and d_{trop_i} are the ionospheric and tropospheric delay for satellite i and ε is the measurement noise. Using UKF, relevant Jacobians for Gain and Covariance prediction, as in equations (1) and (4), are no more calculated, while the non linear measurement function is used without linearization.

Tropospheric corrections are applied to pseudoranges using a standard refraction model (Goad and Goodman model).

Ionosphere propagation error is mitigated through classical Klobuchar method.

For the Doppler measurement, the following expression is used:

$$-\frac{c}{f_{L1}} \cdot D^i = H^i (v^i - v_u) + c \cdot \dot{b} + \varepsilon$$

where D^i is the Doppler measurement for satellite i, obtained from the Carrier Tracking Loop, v^i is the i satellite velocity, H^i is cosine directional vector from the receiver to the satellite i, v_u is the user receiver velocity and ε is the measurement error.

Filter initialization is performed through Bancroft algorithm.

Single satellite measurements are assumed to be uncorrelated. Using such assumption, each measurement can be singularly and sequentially processed, leading to a very significant computational load reduction. In this way, measurements vector size is reduced to 2x1, while low complexity 2x2 matrix inversions operations only have to be performed.

A fine tuning phase has been implemented in order to find suitable values for scaling factors, Q and R covariance elements. Relevant results are described in the following paragraph.

UKF IMPLEMENTATION AND TUNING PHASE FOR GNSS SDR

Starting from the same State and Measurement model used for EKF implementation, UKF has been developed for PVT.

A sensitivity analysis for UKF fine tuning has been performed. Filters parameters choice (Q, R and scaling factors) vary depending on operational conditions. At this aim, different test cases have been performed in static and dynamic conditions in order to determine suitable values for the filter design parameters.

A fixed antenna, with precise position determined through a static geodetic surveying and relevant post-processing, was mounted on the roof of a building and used for static environment tests. Dynamic tests have been performed comparing position, velocity and acceleration behaviors of a receiver mounted on a testing car with the reference ones obtained from a geodetic receiver operating in RTK mode fed by the same antenna.

Q matrix represents the uncertainty in our knowledge about the process. It has a significant impact at steady state, when convergence is achieved. During such phase, K is low and it depends on the initial high Q elements mainly. Higher Q variance elements allow better following acceleration in dynamic conditions, but lead to higher noise and initial overshoot before achieving filter convergence. On the other hand, lower Q variance does not allow a good dynamic tracking, while providing a better damping during the initial convergence. A trade-off between such conditions has to be found.

A sensitivity analysis revealed that Q variances between 1e-5 and 1e-4 are suitable values for static conditions and 0.05-0.1 for dynamic (e.g. in presence of accelerations).

In order to define R matrix values, measurements variances have been derived from real data. Pseudorange error variances have been estimated, taking into account quantization errors and typical pseudorange measurements errors. Variances values in the range of 1000 to 10000 have been considered suitable for a good filter convergence.

Doppler measurements variances have been evaluated analyzing real measurements error dynamics, as reported in Figure 5 for one satellite channel. The range of Doppler measurements variances is therefore selected between 10 and 100.

The State Covariance matrix characterizes the uncertainty about initial conditions. Typical initial State Covariance Matrix variances have been used (100-1000 for position state, 5 for velocity, and literature values for clock bias and drift).

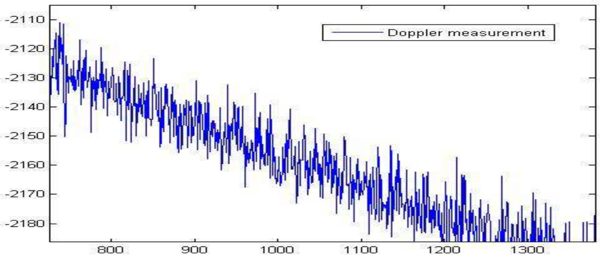

Figure 5 – Doppler measurements error

Taking into account the dynamic model (acceleration is assumed as white noise), Q variances values ranges for the UKF are confirmed from the acceleration analysis performed on real-time test data (see Figure 6). Here, the estimated accelerations (in light blue) are compared to the reference accelerations derived from the geodetic receiver working in RTK mode (in red) connected to the same antenna.

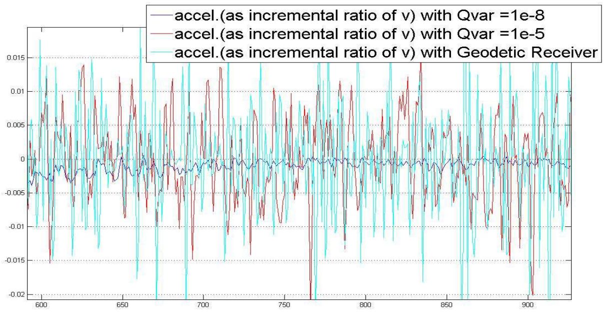

Figure 6 – Acceleration analysis

As reported by the Unscented Kalman Filter theory, such filter is quite insensitive to large initial state errors. It has been demonstrated by real data, as shown in Figure 7, providing an initial position error of 200 m. UKF has demonstrated to be able to converge also with 1000 Km initial errors.

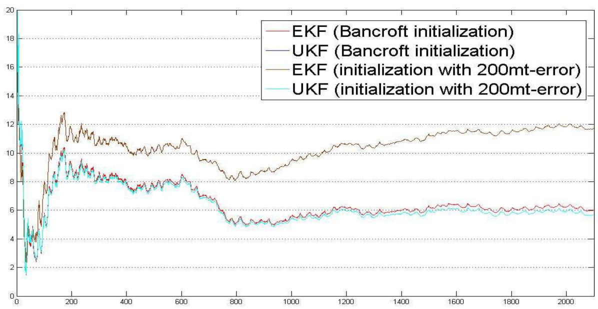

Figure 7 – Initial state error impact

COORDINATES BASED LOCAL AUGMENTATION

For many applications (e.g. ADAS and lane keeping in the road sector), medium accuracy (2m level) at low cost is sufficient. Affordable Augmentation Systems based on low cost GNSS networks infrastructures can improve performances at local level and allow delivery of services for National and Local Public Administrations.

For a GNSS SDR rover, minimization of computational resources is the primary constraint. Furthermore, phase measurement, for performing Carrier smoothing or RTK solutions implementations, can be not directly available in many mass market GNSS receivers and in several GNSS SDR receivers. Therefore, a simple augmentation system based on direct Coordinates Corrections can be useful within this framework.

Coordinates based corrections methods are known since the beginning of GPS applications development (e.g. [R6]).

Considering a Reference Station with known precise coordinates $\bar{x}$, the estimated position $\hat{x}$, affected by an error Δx, can be calculated through Least Mean Square as in the following:

$$\hat{x} = \bar{x} + \Delta x = (H^T H)^{-1} H^T \rho$$

$$\Delta x_{corr} = (H^T H)^{-1} H^T \rho - \bar{x}$$

The estimated error Δx_{corr} can be used as a Local correction for a rover in the neighbors of the Reference Station. Such corrections include impacts of main error sources (e.g. ionospheric delay error) in common between the rover and the Reference Station, provided that same set of satellites are used for position computation.

The corrections can be applied to the rover position as in the following.

$$x_{rov} = \hat{x}_{rov} - \Delta x_{corr} = \bar{x}_{rov} + (\Delta x_{rov} - \Delta x_{corr})$$

where $\bar{x}_{rov}$ is the true rover position, Δx_{rov} is the error in the rover position computation and Δx_{corr} is the applied Coordinate Correction.

In order to respect the constraint of using same satellites for Reference Stations and Rover position calculation, as required by classical Coordinates Corrections algorithms, several corrections broadcasting methods can be implemented.

For the sake of our investigations, the following two common Augmentation architectures have been taken into account:

- **Architecture 1**: the rover transmits satellites used for PVT to the GNSS Reference Station Network Control Centre; it calculates relevant corrections and transmit them to the rover

- **Architecture 2**: the GNSS Reference Station Network Control Centre calculates the whole set of Coordinates Corrections for each possible satellites combination and broadcast them to the rover

Both the above solutions imply the availability of computational resource and the implementation of dedicated solutions at GNSS Reference Station Network Control Centre side. While the Architecture 1 implies the availability of a bidirectional communication channel, the Architecture 2 broadcasting solution requires significant computational resource (e.g. 386 combinations solutions to be processed at each epoch for 10 satellites in visibility at Reference Station side and 6 to 10 satellites of them visible at rover side).

Therefore, a third architecture has been designed and developed.

Currently, RTCM 2.3 and 3.0 standard messages do not include Coordinate Corrections transmission. An adaptation of the Coordinate Corrections method to the available messages is needed for implementing such Augmentation system. The used Architecture is reported in Figure 8.

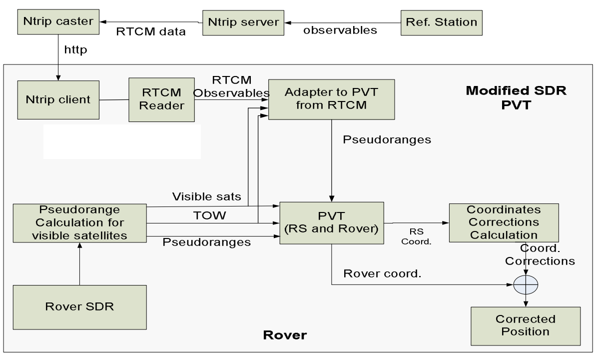

Figure 8 – Coordinate Corrections method Architecture

Reference Station pseudorange measurements for all visible satellites are broadcasted using NTRIP protocol to the SDR rover using RTCM 2.3 #19 messages (or RTCM 3.0 message 1004). Pseudorange measurements are read and converted into the data format used by PVT through an Adapter module.

Using such architecture, the only needed modifications are implemented on the rover PVT. Reference Station position estimation and corrections calculation is performed by the rover using the same rover PVT module. Only satellites in common visibility between the rover and the Reference Station, with relevant ephemeris extracted by the rover PVT, are used. In that way, there is not the need to calculate Reference Station position and relevant Corrections for all possible combinations of in common satellites.

Once calculated, Coordinates Corrections are applied to the rover PVT estimation in order to produce a corrected position in NMEA format.

In order to estimate the spatial decorrelation of corrections, a post-processing sensitivity analysis has been performed. Precise rover position, calculated through post-processing analysis, has been compared with the corrected position using Coordinate Corrections calculated from several Reference Stations spread around the rover.

It has been derived that corrections maintains their validity at least up to 100 Km of separation, as reported in Figure 9, where Grosseto Reference Station is 90 Km far from the rover position.

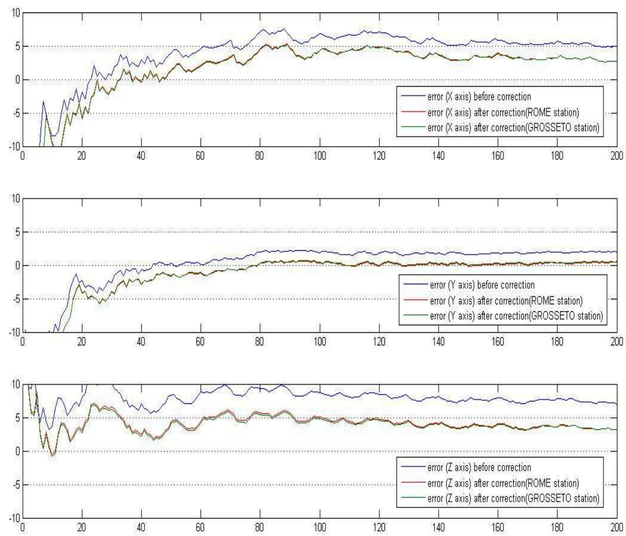

Figure 9 – Differential Correction post-processing analysis (ROMA and GROS Reference Stations)

The above architecture has been developed and tested using raw measurements coming from existing Reference Stations Networks as Local Augmentation System. Pseudoranges calculated by the Reference Station are dependent on embedded models used for errors detection and mitigation (e.g. ionospheric corrections, tropospheric models or carrier smoothing). Therefore pseudorange at Reference Station and rover side are not directly comparable ad can lead to slight mismatches in the corrections.

A Local Augmentation system usually implies high installation and maintenance costs. A system totally based on SDR, both for rover and Reference Stations, could lead to significant costs savings and simplifications for the implementation of medium accuracy services and systems spread over a wide territory. Furthermore, such an implementation will guarantee homogenous service levels and raw measurements quality, due to the use of same error models and PVT algorithms at rover and Reference Station side.

Future activities of our group will work around such solution.

RESULTS AND PERFORMANCES

A testing car has been set-up in order to evaluate UKF performances. A single antenna is able to feed both a GNSS geodetic receiver operating in RTK mode and the GNSS SDR through an antenna splitter developed for this purpose (see Figure 10).

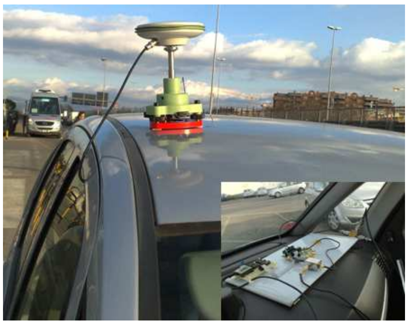

Figure 10 – Testing Car

A reference trajectory has been followed (as shown in Figure 11), performing change of directions and acceleration/decelerations. Precise RTK positions have been logged in the geodetic receiver memory to be used as reference trajectory, while SDR EKF and UKF outputs have been logged in a file for post-processing.

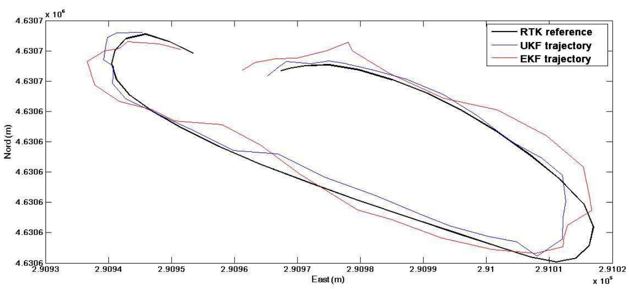

Figure 11 – Dynamic test trajectory

Relevant results at steady state for a dynamic test are reported in Figure 12, compared to classical EKF implementation. After an initial static phase (first tenth of s), the car starts moving and performs two curves with relevant change of directions and deceleration/acceleration. At the end, a new static phase is implemented.

As can be seen, with reference to the true trajectory, UKF is quite robust during the whole trajectory and it is resistant to rapid change of directions and in presence of accelerations, as well as to satellite geometry change due to the shadowing of the surrounding buildings, occurring at the beginning and at the end of the route. On the contrary, EKF does not react promptly to such dynamic conditions, leading to a significant error increase.

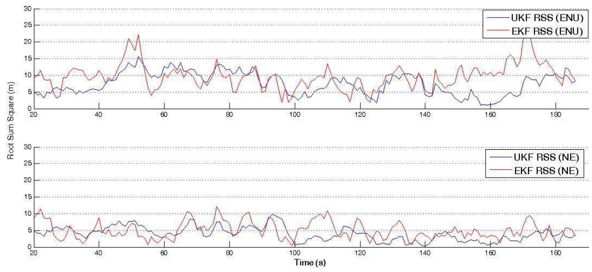

Figure 12 – UKF dynamic results

Concerning the Coordinates Corrections method, a static test has been carried on a point with known coordinates. The RSS (Root Sum Square) planimetric error and the 3D RSS error before and after corrections application are reported in Figure 14 and Figure 14.

Under nominal ionospheric conditions, a final accuracy improvement of at least 60% is envisaged using such method. A 3D RMS error 2.3 m has been achieved applying coordinates corrections, against about 9.1 m for absolute positioning.

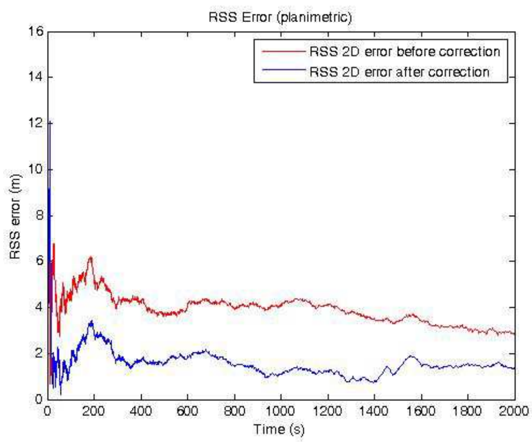

Figure 13 – Error before and after corrections (2D)

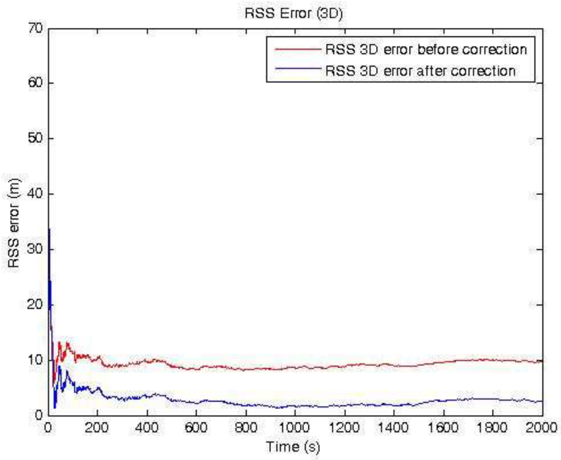

Figure 14 – Errors before and after corrections (3D)

CONCLUSIONS

A scaled Unscented Kalman filter has been designed and implemented within the SDR PVT module of the Sogei's GNSS SDR in order to overcome classical limitations of an Extended Kalman Filter. UKF does not make use of linearization and performs Covariance Matrix propagation by points (e.g. sigma points). It allows a better mean and Covariance matrix propagation during abruptly changing routes in dynamic situations.

Relevant UKF parameters (Q variances, P variances, UKF scaling parameters) have been selected after a fine tuning phase.

The paper demonstrated through on-field test that a significant improvement in robustness and in position and velocity error estimation is achieved by UKF in dynamic situations with respect to classical EKF.

Further improvements are under investigation and can be achieved through the development of automatic tuning methods for Q matrix elements.

A simple Local Augmentation system using Coordinate Corrections has been developed for SDR based applications. The GNSS SDR rover receiver pseudorange measurements are transmitted by a geodetic Reference Station of a GNSS Network using standard RTCM 2.3 messages and NTRIP protocol. An improvement in the order of 60% in position estimation is envisaged.

Such results can pave the way for a possible implementation of low cost Local Augmentation systems totally based on SDR receivers (both rover and Reference Station side).

Applications like ADAS and lane keeping can beneficiate from such method and will be further investigated by the Authors in the following years.

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